FIN 532

Problem Set 3

①

a. The covariance of the fund returns is

$\sigma_{S,B} = 0.3 \times 0.15 \times 0.10$

$$w_B = \frac{\sigma_S^2 - \sigma_{S,B}}{\sigma_S^2 + \sigma_B^2 - 2\sigma_{S,B}} = \frac{19}{23} \approx 82.61\%$$

$$w_S = \frac{4}{23} \approx 17.39\%$$

b. $\mu_{MV} = \frac{19}{23} \times 0.12 + \frac{4}{23} \times 0.20 = 13.39\%$

$$\sigma_{MV} = \sqrt{w_B^2 \sigma_B^2 + w_S^2 \sigma_S^2 + 2 w_B w_S \sigma_{B,S}}$$

$= 13.92\%$

c.

w_S	w_B	μ_P	σ_P
0%	100%	12%	15%
20%	80%	13.6%	13.94%
40%	60%	15.2%	15.70%
60%	40%	16.8%	19.53%
80%	20%	18.4%	24.48%
100%	0%	20%	30%

(2) The variance of a portfolio investing $1-w_B$ in A and w_B in B is

$$\sigma_p^2 = \left((1-w_B)\sigma_A - w_B\sigma_B\right)^2$$

which can be made 0 if

$$(1-w_B)\sigma_A = w_B\sigma_B$$

$$\sigma_A = w_B(\sigma_A + \sigma_B)$$

$$w_B = \frac{\sigma_A}{\sigma_A + \sigma_B} = \frac{5}{15} = \frac{1}{3}$$

$$w_A = \frac{2}{3}$$

$$\Rightarrow r_F = \mu_P = \frac{2}{3} \times 0.10 + \frac{1}{3} \times 0.15 = 11.67\%$$

(3) Certainly the standard deviation of returns and the correlation with returns of other assets.

(4) Here alternative i. is true. Alternative ii is false since it's possible that the expected return of MVP might be lower than the risk-free rate. The MVP can never be the optimal tangency portfolio, but it will include all risky securities.

(5) The statement is true if all investors have preferences like

$$U = \mu - \frac{1}{2} A \sigma^2 .$$

This is the essence of the CAPM.

(6) This is not true in general. We saw in class that covariances between the returns of risky assets also matter.

(7) a. Let's call P the original portfolio and Q the new portfolio.

$$r_Q = w_P \, r_P + w_{ABC} \, r_{ABC}$$

$$w_P = \frac{1,400,000}{1,600,000} = 0.875$$

$$w_{ABC} = 0.125$$

$$\mu_Q = 0.875 \times 0.092 + 0.125 \times 0.16 = 10.05\%$$

$$\sigma_Q^2 = 0.875^2 \times 0.08^2 + 0.125^2 \times 0.16^2$$
$$+ 2 \times 0.875 \times 0.125 \times 0.08 \times 0.16 \times 0.4$$

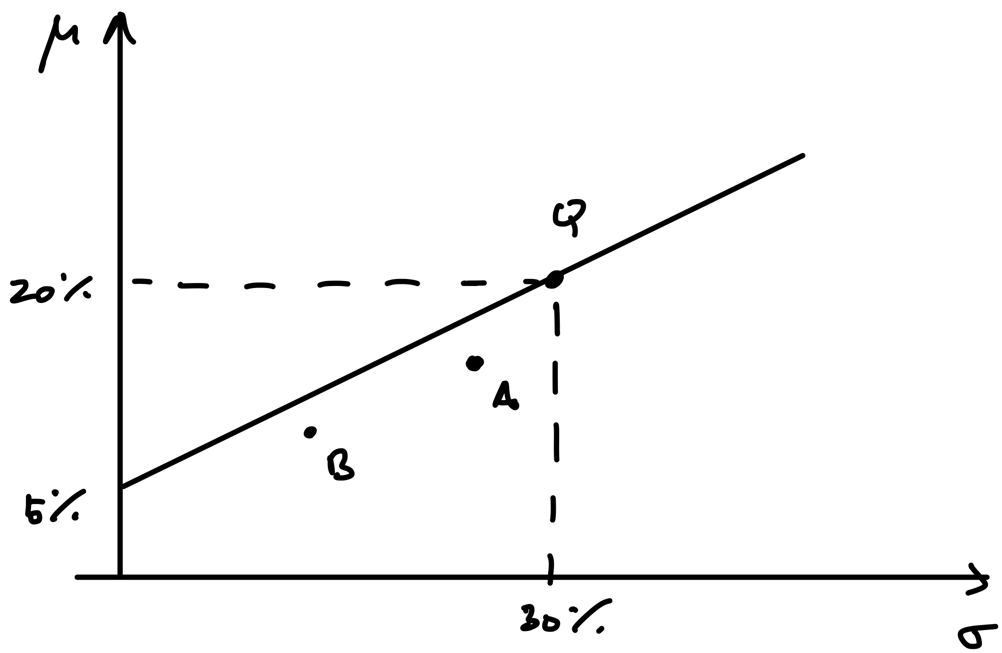
$$\sigma_Q = \sqrt{\sigma_Q^2} = 8.01\%$$

b. $\mu_Q = 0.875 \times 0.092 + 0.125 \times 0.06 = 8.80\%$

$\sigma_Q = 0.875 \times 0.08 = 7\%$

c. The comment is incorrect since the correlation of XYZ with her original portfolio need not be 0.4. The diversification benefits of XYZ might be different.

8 a.



$SR_Q = \dfrac{20-5}{30} = \dfrac{1}{2}$ $\quad SR_A = \dfrac{15-5}{25} = \dfrac{2}{5} = 0.4 < \dfrac{1}{2}$

$SR_B = \dfrac{8-5}{15} = \dfrac{3}{15} = \dfrac{1}{5} = 0.2 < \dfrac{1}{2}$

Only Q is efficient since the Sharpe ratios of A and B are lower.

b. We can combine r_f and Q to obtain a 30% return.

$$0.3 = (1-w)\,0.05 + w \times 0.20$$

$$w = \frac{0.3 - 0.05}{0.2 - 0.05} = \frac{0.25}{0.15} = \frac{3}{5} = 60\%$$

We should invest 60% in Q and 40% in the risk-free asset.
The standard deviation of such portfolio is

$$\sigma = 0.6 \times 0.3 = 18\%$$

c. $$0.1 = (1 - w_B)\,0.15 + w_B \times 0.08$$

$$w_B = \frac{0.1 - 0.15}{0.08 - 0.15} = \frac{5}{7}$$

$$w_A = \frac{2}{7}$$

$$\sigma^2 = \left(\frac{2}{7}\right)^2 \times 0.25^2 + \left(\frac{5}{7}\right)^2 \times .15^2$$
$$+ 2 \times \left(\frac{2}{7}\right) \times \left(\frac{5}{7}\right) \times 0.25 \times 0.15 \times 0.24$$

$\sigma = 14.23\%$ $SR = \dfrac{10 - 5}{14.23} = 0.3513 < \frac{1}{2}$

$\Rightarrow$ not efficient.

(4) a. $\sigma_{AB} = \sqrt{0.01} \times \sqrt{0.0625} = 0.025$

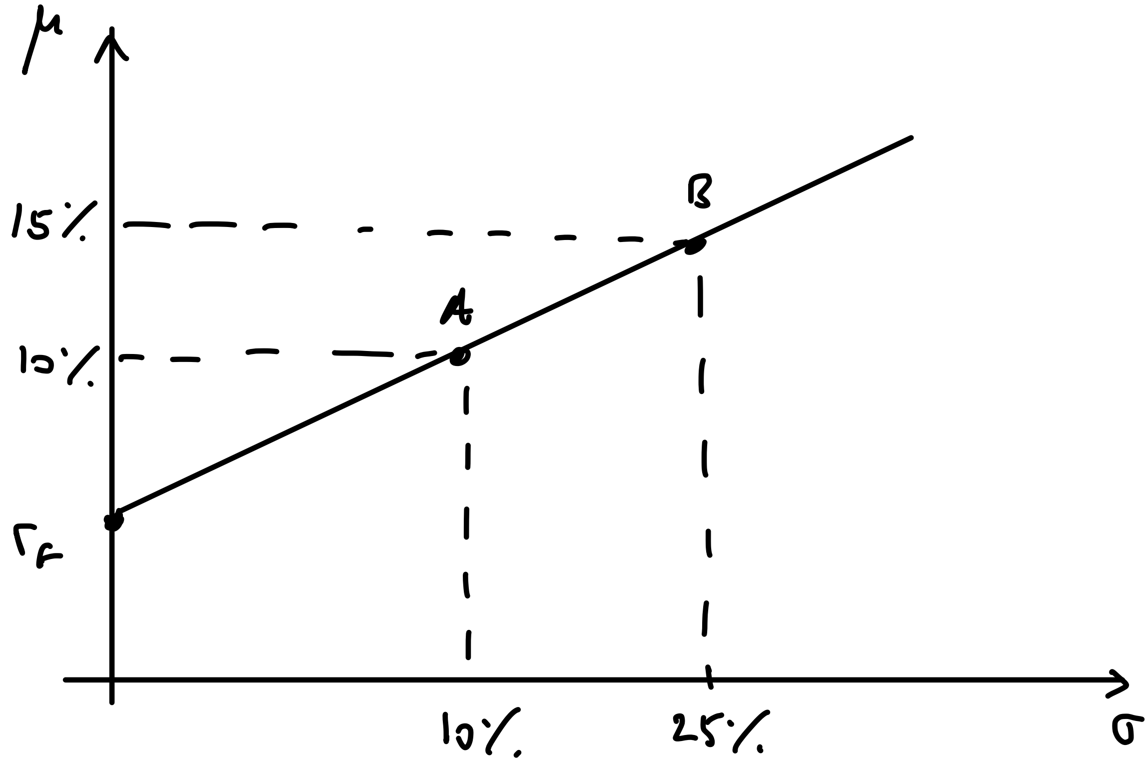
b. $0.05 = (1 - w_B) \times 0.10 + w_B \times 0.15$

$$w_B = \frac{0.05 - 0.10}{0.15 - 0.10} = -1$$

$w_A = 2.$

You should short-sell 100% of B and invest your capital plus the proceeds of your short selling in A.

c. The MVP will have zero risk.



$$\frac{0.1 - r_F}{0.1} = \frac{0.15 - 0.10}{0.25 - 0.10} = \frac{1}{3}$$

$$r_F = 0.1 - \frac{1}{3} \times 0.1 = 6.67\%$$

d. Both assets A and B are efficient since they have the same Sharpe ratio which in this case is the maximum. The portfolio computed in b. is inefficient since its expected return is less than the risk free rate.

e. We know that the portfolio that achieves 6.67% has zero risk. We can compute its weights as

$$(1-w_B)\sigma_A + w_B\sigma_B = 0$$

$$w_B(\sigma_B - \sigma_A) = -\sigma_A$$

$$w_B = -\frac{\sigma_A}{\sigma_B - \sigma_A} = -\frac{0.1}{0.25 - 0.1} = -\frac{2}{3}$$

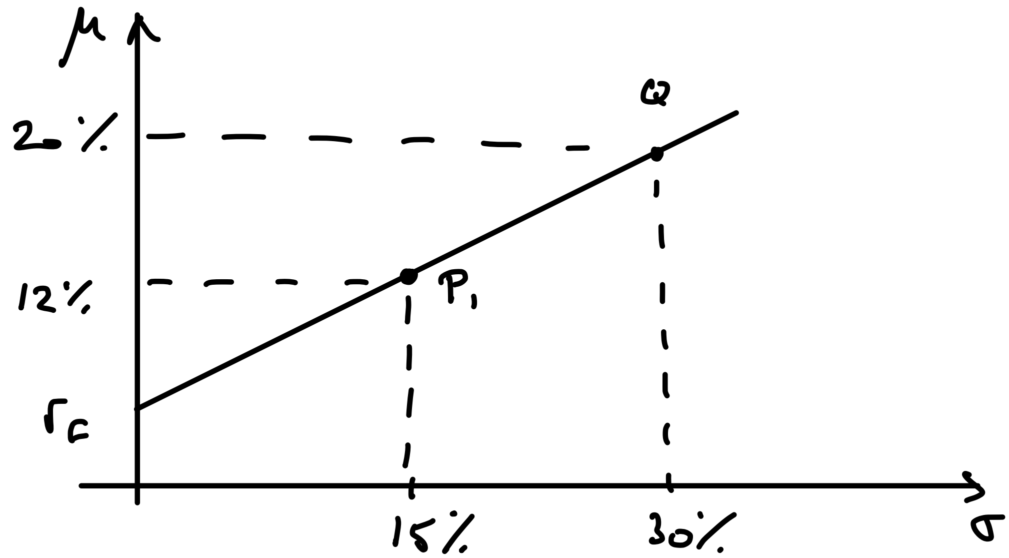
$$w_A = 1 + \frac{2}{3} = \frac{5}{3}$$

<u>Check</u>: $\mu = \frac{5}{3} \times 0.10 - \frac{2}{3} \times 0.15 = 6.67\%$

Therefore, borrow as much as you can at 4%. Short sell 2/3 of that amount of asset B. Invest all that in asset A.

(10)

a. Since P_1 is optimal, it is in the CAL of Q.



Thus, $\dfrac{0.12 - r_F}{0.15} = \dfrac{0.20 - 0.12}{0.30 - 0.15}$

$\Rightarrow \quad 0.12 - r_F = 0.08$

$r_F = 4\%$

b. $0.15 = w \times 0.30 \Rightarrow w = 0.5$

So Peter invests 50% in the risk free asset and 50% in Q.

c. $0.4 = w \times 0.3 \Rightarrow w = \dfrac{4}{3} \qquad 1 - w = -\dfrac{1}{3}$

He should borrow 33.33% of his wealth at the risk-free rate and invest 133.33% in Q.

$\mu_2 = -\dfrac{1}{3} \times 0.04 + \dfrac{4}{3} \times 0.2 = 25.33\%$

(11) a. $Er_x = 0.05 + 0.7 \times (0.15 - 0.05)$

$= 12\%$

$\alpha_x = 15\% - 12\% = 3\%$

$Er_y = 0.05 + 1.6\,(0.15 - 0.05)$

$= 21\%$

$\alpha_y = 18\% - 21\% = -3\%$

b. $SR_x = \dfrac{12 - 5}{37} = 0.189$

$SR_y = \dfrac{21 - 5}{26} = 0.615$

c. i. Stock x has higher α so it is more appropriate to add to a well diversified portfolio.

ii. Stock Y has higher SR so it is more appropiate to hold as a single stock portfolio.

d. $Er_x = \alpha_x + E^{CAPM} r_x = 15\%$

$\sigma_x = 0.7 \times 0.16 = 11.2\%$

$Er_y = \alpha_y + E^{CAPM} r_y = 18\%$

$\sigma_y = 1.6 \times 0.16 = 25.6\%$